Modular geometry of complex time: topological origin of the inflationary α-attractor parameter

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We propose a framework in which physical time is necessarily a complex variable $\tau \in \mathbb{H}^2$ (the upper half-plane) and the photon is topologically an elliptic curve with modular parameter τ. Both statements follow from a single meta-postulate of structural realism (M0): physical observables are functions on isomorphism classes, not on representatives. Complex time is *not* postulated but derived—a theory describing the *structure* of rotation, rather than individual rotations, requires at minimum a complex variable, and $\mathbb{C}$ is the unique minimal field carrying that structure. Modular invariance under $\mathrm{PSL}(2,\mathbb{Z})$ then follows as a theorem. The inflationary target space is strictly the Poincaré half-plane with constant Gaussian curvature $K = -1$, forcing the α-attractor parameter to the topological invariant $\alpha = 2/3$. This yields a parameter-free consistency relation for the tensor-to-scalar ratio, $r = 2(1-n_s)^2$, *given* the T-model effective potential (Branch A, Sec. V.1). Inserting the Planck 2018 value $n_s = 0.9649 \pm 0.0042$ gives $r \approx 0.0025$ with $N_e \approx 57$, testable by LiteBIRD and distinguishable from the Starobinsky R^2 model ($r \approx 0.0037$). If instead the potential itself is required to be fully $\mathrm{PSL}(2,\mathbb{Z})$-invariant (Branch B, Sec. V.2), the plateau is approached doubly-exponentially and r is suppressed to $O(10^{-5})$ while $n_s \simeq 1 - 2/N_e$ is preserved; the T-model is not derived from this branch (modular averaging is ruled out, Sec. V.2), so Branch A is an independent assumption, and LiteBIRD/CMB-S4 are exactly the experiments that discriminate between the two. A final section records three speculative extensions—stellar systems as τ-states, a neutrino–neutron temporal symmetry, and Maxwell's equations as a real projection—each explicitly flagged as conjectural and outside the parameter-free core.

I. INTRODUCTION

The α-attractor framework [1, 2] provides a unified description of slow-roll inflation on negatively curved field spaces. The inflaton φ lives on a hyperbolic manifold with field-space curvature $K_{\text{fields}} = -2/(3\alpha)$, and the leading-order predictions depend on a single parameter α:

$$n_s = 1 - \frac{2}{N_e}\,, \qquad r = \frac{12\alpha}{N_e^2}\,, \tag{1}$$

with N_e the number of e-folds. For $\alpha = 1$ one recovers the Starobinsky R^2 model [3].

Despite the elegance of the construction, α remains a free parameter in all existing implementations: it is either fit to data or selected by embedding in string/M-theory compactifications, where discrete values $3\alpha \in \{1, 2, 3, 7\}$ arise [4]. None of these selections is *derived* from first principles.

In this article we show that a single meta-postulate—structural realism applied to the concept of time—fixes $\alpha = 2/3$ as a topological invariant of the modular curve. The resulting prediction $r = 2(1-n_s)^2$ contains no free parameter. The logical chain is

$$\begin{aligned} &\mathrm{M0} \;\Rightarrow\; \tau \in \mathbb{H}^2 \;\Rightarrow\; \text{photon} = \mathcal{E}_\tau \;\Rightarrow\; \mathrm{PSL}(2,\mathbb{Z}) \\ &\quad\Rightarrow\; K = -1 \;\xRightarrow{+\mathrm{A4}}\; \alpha = \tfrac{2}{3}. \end{aligned}$$

The final step requires one further explicit postulate, A4 (Sec. IV.3): that the inflaton field space *is* the complex-time plane.

II. FRAMEWORK

A. Meta-postulate M0 (structural realism)

Physical observables are functions on isomorphism classes of physical states, not on their representatives. If two configurations differ only by an isomorphism of their internal structure, no physical measurement distinguishes them. This is the meta-assumption already underlying gauge theory, where observables factor through gauge orbits.

B. From M0 to complex time

A theory of time must account not only for individual temporal processes (rotations, oscillations, phase evolution) but for the *structure of rotation itself.* A real parameter $t \in \mathbb{R}$ suffices to label a single rotation; structural realism demands instead a description of the isomorphism class of rotational structures. Two rotations related by a conformal equivalence are structurally identical yet carry distinct real parameterizations, so a single real number cannot be the fundamental datum.

Lemma 1 (uniqueness of $\mathbb{C}$). $\mathbb{C}$ is the unique minimal field supporting a modular structure over rotations.

Proof. The candidate fields are $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{H}$. (i) $\mathbb{R}$ is insufficient: it carries no intrinsic rotational automorphisms and no conformal structure; observables are real-valued, but the structure from which they are projected need not be—exactly as quantum-mechanical probabilities are real projections $|\psi|^2$ of complex amplitudes.

(ii) $\mathbb{H}$ is excessive: quaternions are non-commutative, so holomorphic function theory does not extend to $\mathbb{H}$, and the moduli space of elliptic curves is defined over $\mathbb{C}$, not over $\mathbb{H}$; no quaternionic analogue of $\mathbb{H}^2/\mathrm{PSL}(2,\mathbb{Z})$ exists. (iii) $\mathbb{C}$ is the unique field that is simultaneously closed under arithmetic, algebraically closed, and naturally equipped with a conformal structure making rotations into automorphisms. □

Therefore M0 applied to temporal evolution *requires* the fundamental time variable to be complex,

$$\tau \in \mathbb{H}^2 = \{\tau \in \mathbb{C} : \mathrm{Im}\,\tau > 0\}\,, \tag{2}$$

the restriction to the upper half-plane following from the physical requirement that partition functions converge and norms be positive-definite. Real-valued observables are projections, not the fundamental layer—mirroring quantum mechanics, where $\mathbb{C}$-valued amplitudes are primary and $\mathbb{R}$-valued probabilities derived. Complex time is thus not an axiom but the unique necessary structure for a theory that describes rotation *structurally* rather than parametrically.

C. From complex time to photonic topology

Given a single complex modulus τ, we ask which fundamental object is determined by τ up to conformal equivalence. By the uniformization theorem and the classification of Riemann surfaces, a surface carrying exactly one complex modulus has genus $g = 1$: genus $g = 0$ surfaces have no moduli, and genus $g \geq 2$ surfaces have $3g - 3 \geq 3$ moduli. Only genus-1 surfaces—tori—have a moduli space of dimension exactly one over $\mathbb{C}$.

Identifying this genus-1 object with the photon is not arbitrary. The photon is the irreducible mediator of all measurement, interaction and information transfer: it carries energy, momentum, polarization and phase, yet has no mass, no charge and no composite internal structure. In the hierarchy of physical objects it occupies precisely the place the elliptic curve occupies among Riemann surfaces—the simplest object with nontrivial internal structure (one complex modulus). Its $U(1)$ gauge freedom corresponds to one S^1 factor of the torus $S^1 \times S^1$; the second S^1 carries the internal phase cycle parameterized by $\mathrm{Im}\,\tau$. The photon is therefore topologically

$$\mathcal{E}_\tau \;\cong\; \mathbb{C}/(\mathbb{Z} + \mathbb{Z}\tau)\,, \tag{3}$$

an elliptic curve with complex structure τ.

D. Observer projection

A classical observer does not have access to the global complex phase; it extracts only the $\mathrm{Stab}(i)$-invariant of the modular geometry. Along the canonical trajectory $\mathrm{Re}\,\tau = 0$ this reduces to the signed hyperbolic distance from the elliptic fixed point $\tau = i$,

$$\varphi(\tau) = \ln \tau_2\,, \qquad \tau_2 \equiv \mathrm{Im}\,\tau\,, \tag{4}$$

which is exactly the canonically normalized inflaton of Sec. V. (The naive statement "the observer measures $\mathrm{Re}\,\tau$" is not $\mathrm{PSL}(2,\mathbb{Z})$-invariant and is ill-defined under M0 away from this trajectory; it is retired here in favor of the $\mathrm{Stab}(i)$ formulation, which is what the rest of this paper actually uses.)

III. MODULAR INVARIANCE AS A THEOREM

Theorem 1. Given M0, complex time $\tau \in \mathbb{H}^2$, and photonic topology, physical observables are invariant under $\mathrm{PSL}(2,\mathbb{Z})$.

Proof sketch. The photon is an elliptic curve $\mathcal{E}_\tau$ parameterized by $\tau \in \mathbb{H}^2$. By a classical theorem of the theory of modular forms [8], the moduli space of elliptic curves over $\mathbb{C}$ is $\mathbb{H}^2/\mathrm{PSL}(2,\mathbb{Z})$: two parameters τ, τ' yield isomorphic curves if and only if they lie in the same $\mathrm{PSL}(2,\mathbb{Z})$-orbit. By M0, observables factor through isomorphism classes; hence observables are $\mathrm{PSL}(2,\mathbb{Z})$- invariant functions on $\mathbb{H}^2$. □

The group is not postulated: it arises inevitably from the automorphism structure of the moduli space forced by the photonic identification.

IV. DERIVATION OF $\alpha = 2/3$

A. Poincaré metric and $K = -1$

The unique Riemannian metric on $\mathbb{H}^2$ invariant under $\mathrm{PSL}(2,\mathbb{R}) \supset \mathrm{PSL}(2,\mathbb{Z})$ is the Poincaré metric

$$ds^2 = \frac{dx^2 + dy^2}{y^2}\,, \tag{5}$$

with constant Gaussian curvature $K = -1$.

B. Area of the fundamental domain

The fundamental domain $\mathcal{F}$ of $\mathrm{PSL}(2,\mathbb{Z})$ has hyperbolic area

$$\mathrm{Area}(\mathcal{F}) = \frac{\pi}{3}\,, \tag{6}$$

from Gauss–Bonnet applied to the modular orbifold $\mathcal{X}_1$ with $\chi(\mathcal{X}_1) = -1/6$.

C. Fixing α

The α-attractor field-space curvature is $K_{\mathrm{fields}} = -2/(3\alpha)$. Given the Poincaré metric fixed above ($K =$

-1), the value of α follows immediately *if* the inflaton field space is identified with the complex-time plane $\mathbb{H}^2$ itself, i.e. $K_{\text{fields}} \equiv K$. This identification is an explicit, independent physical postulate (A4): the same geometric object that carries the structure of complex time also carries the inflaton kinetic term. It is not derived from M0, Theorem 1, or the normalization $K = -1$ alone — it is the minimal assumption under which the modular-curve geometry of Secs. II–IV has any bearing on inflationary observables at all; without it τ would remain a purely kinematic time variable disconnected from (n_s, r). Of the postulates in this framework, A4 carries the least independent motivation: it is introduced for its predictive economy, not derived, and a completed microscopic derivation of the effective action in Sec. V from τ-dynamics would either justify it or replace it — this remains open. Granting A4 and equating $K_{\text{fields}} = K = -1$,

$$-1 = -\frac{2}{3\alpha} \implies \alpha = \frac{2}{3} = \frac{2\,\text{Area}(\mathcal{F})}{\pi} = -4\chi(\mathcal{X}_1)\,. \quad (7)$$

This is a topological invariant of the modular curve: it cannot be changed by smooth deformations preserving the orbifold topology.

D. Parameter-free prediction

Eliminating N_e from (1) with $\alpha = 2/3$,

$$\boxed{r = 2\,(1 - n_s)^2} \quad (8)$$

For $n_s = 0.9649 \pm 0.0042$ (Planck 2018 [9]),

$$r \approx 0.0025\,, \qquad N_e \approx 57\,. \quad (9)$$

V. EFFECTIVE ACTION AND FORMAL DERIVATION

The complete $\mathrm{PSL}(2,\mathbb{Z})$-invariant effective action on the τ-plane is

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\text{Pl}}^2}{2}R - \frac{(\partial\tau_1)^2 + (\partial\tau_2)^2}{2\tau_2^2} - V(\tau)\right]. \quad (10)$$

During inflation the trajectory descends along the imaginary axis from the cusp at $i\infty$, so $\tau_1 \approx$ const and $\partial\tau_1 \approx 0$; the kinetic term reduces to $L_{\text{kin}} = -\frac{1}{2}(\partial\tau_2)^2/\tau_2^2$. Defining the canonical field $\varphi = \ln\tau_2$ normalizes the kinetic term exactly to $-\frac{1}{2}(\partial\varphi)^2$.

A. T-model effective potential (Branch A)

The kinetic sector fixed by Theorem 1 is fully $\mathrm{PSL}(2,\mathbb{Z})$-invariant ($K = -1$); the potential sector need not be, and — as Sec. V B below shows — cannot consistently be taken as the naive fully modular-invariant candidate. The simplest potential compatible with the reformulated observer projection of Sec. II.4, which commits only to the residual $\mathrm{Stab}(i) = \langle S : \tau \to -1/\tau\rangle$ subgroup rather than the full modular group, is the standard α-attractor T-model in canonical variables,

$$V(\varphi) = \Lambda^4 \tanh^2(\varphi/\sqrt{6\alpha})\,. \quad (11)$$

For $\alpha = 2/3$ this is $V(\varphi) = \Lambda^4\tanh^2(\varphi/2)$, with the coefficient 1/2 fixed arithmetically by $\alpha = 2/3$ alone — i.e. by Theorem 1 and A4, with no further free choice. A numerical slow-roll integration of this potential gives, for $N_e = 57$, $n_s = 0.965$ and $r = 0.0024$, reproducing Eq. (8) to within the few-percent slow-roll accuracy. This potential is accepted here as the effective theory selected by the observer projection; it is *not* itself derived from a fully modular-invariant microscopic action, as the next subsection makes precise.

B. Fully modular-invariant potential (Branch B)

One might instead demand that the potential itself, not only the kinetic term, be invariant under the full modular group, e.g.

$$V(\tau) = \Lambda^4\tanh^2\!\left(\frac{H(\tau)}{2}\right), \qquad H(\tau) = -\ln\left[\tau_2\,|\eta(\tau)|^4\right], \quad (12)$$

with η the Dedekind eta function. Near the cusp, $\ln|\eta(\tau)| \to -\pi\tau_2/12$, so

$$H(\tau) \;\simeq\; \frac{\pi}{3}\tau_2 - \ln\tau_2 \;=\; \frac{\pi}{3}e^{\varphi} - \varphi\,, \quad (13)$$

which is dominated by the term *exponential* in the canonical field $\varphi = \ln\tau_2$, not by φ itself: $H(\tau)$ does *not* reduce to φ near the cusp. Any potential $V = F(H)$ therefore approaches its plateau doubly-exponentially in φ, a behavior analyzed independently by Kallosh and Linde in $\mathrm{SL}(2,\mathbb{Z})$ cosmology [5–7]. Standard slow-roll analysis of this branch gives

$$n_s \simeq 1 - \frac{2}{N_e}\,, \qquad r \simeq \frac{8}{x_*^2 N_e^2}\,, \qquad e^{x_*} = \frac{12}{\pi}x_*^3 N_e\,, \quad (14)$$

so $x_* \approx 13.1$ for $N_e = 57$ and

$$r_B \approx 1.4{\times}10^{-5}\;(N_e = 57)\,, \qquad r_B \approx 7{\times}10^{-6}\;(N_e = 78)\,, \quad (15)$$

six orders of magnitude below the T-model prediction and undetectable by LiteBIRD or CMB-S4, while n_s is unchanged. An explicit attempt to derive the Branch A T-model from this fully modular-invariant potential by averaging over τ_1 was carried out and **ruled out**: plain modular averaging cannot convert the doubly-exponential $H(\tau)$ into the linear canonical argument φ that the T-model requires. Branch A is therefore an

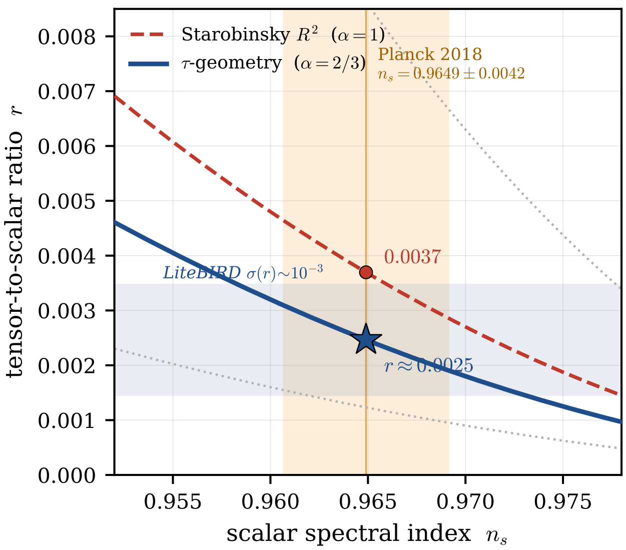

FIG. 1. Predictions in the (n_s, r) plane. The τ-geometry line $r = 2(1 - n_s)^2$ (solid blue) is shown with the Starobinsky R^2 model (red dashed) and Kallosh–Linde benchmarks (dotted gray). The star marks the prediction at $n_s = 0.9649$ (Planck 2018); the orange band is the Planck 2018 1σ range on n_s. The shaded blue band indicates the projected LiteBIRD sensitivity $\sigma(r) \sim 10^{-3}$.

Model	α	3α	r ($n_s = 0.965$)
τ-geometry	**2/3**	**2**	**0.0025**
Starobinsky R^2	1	3	0.0037
KL ($3\alpha = 7$)	7/3	7	0.0086
KL ($3\alpha = 1$)	1/3	1	0.0012

TABLE I. Comparison of α-attractor models. The τ-geometry value $3\alpha = 2$ is a topological invariant of the modular curve, unlike specific string-landscape compactifications.

independent assumption — the simplest potential compatible with the observer projection — not a derived limit of Branch B. Distinguishing the two branches is exactly the observational test of Sec. VI: Branch A predicts $r \approx 0.0025$, Branch B predicts r unobservably small.

VI. MODEL COMPARISON AND TESTABILITY

The decisive observational test is τ-geometry versus Starobinsky, with $\Delta r \approx 0.0012$ (Fig. 1, Table I). LiteBIRD [10] will reach $\sigma(r) \sim 10^{-3}$, providing $\sim 1\sigma$ discrimination from Starobinsky on its own; a CMB-S4-class successor [11] would sharpen this to $\sim 2\sigma$, but CMB-S4 itself lost DOE/NSF funding in July 2025 [12], so that sharper discrimination is contingent on a yet-unfunded successor program, not on an active one. The framework is falsified if $r > 0.01$ or $r < 0.0005$ ($\alpha = 2/3$ excluded), if $r \approx 0.004$ (Starobinsky preferred), or by a formal error in the chain M0 $\to$ complex time $\to K = -1 \to$ (A4) $\to \alpha = 2/3$, or by a principled argument that A4 cannot hold.

VII. SPECULATIVE EXTENSIONS AND OUTLOOK

The remainder is conjectural. None of the following is parameter-free, none is promoted to an axiom, and none modifies the core result (8). Each proposal is stated together with the specific gap that must be closed before it could be regarded as established. They are recorded here because they are, in principle, falsifiable, and because the same complex-projection logic that fixes α suggests them.

The unifying picture is summarized in Fig. 3: the imaginary axis $\mathrm{Im}\,\tau$ carries the bulk of the structure (~96%, gravitationally felt as dark sectors), while only the $\operatorname{Re}\tau$ projection (∼4%) is directly observed, the transition acting as a tanh-shaped filter. Figure 2 renders the same intuition artistically: a collapsed $2i$ node (left), interlinked tori with their wave projections, and the generational ladder.

A. Stellar systems as τ-states

A star together with its planetary system may be read as one irreducible state in τ-space, with orbits, radiation and gravity as decompositions of that object into observables along different slices of $\mathbb{H}^2$. In this reading the star is a local elliptic anchor (a saddle of the modular height), and a supermassive black hole is a degenerate "$2i$" configuration in which the real projection collapses—an event horizon being precisely the locus where $\operatorname{Re}\tau$ ceases to be defined. A naive attempt to read the cosmic baryon fraction $\Omega_b \approx 0.05$ off the $\tanh^2$ filter at the end of inflation fails quantitatively ($\tanh^2(\varphi_{\rm end}/2) \approx 0.27$, not 0.05); the proposal survives only if a principled evaluation point or ratio is identified. The three-generation count is *not* claimed as an independent consequence here—it is, at best, a re-reading of the same modular fixed-point structure and is kept out of the parameter-free core deliberately.

B. Neutrino–neutron temporal symmetry

The neutron decays in $\operatorname{Re}\tau$ with $t_{1/2} \approx 880\,$s; one may read the neutrino as the $\operatorname{Im}\tau$ image of the same transition—not a decay but a condensation, formally a sign reversal of $d\operatorname{Im}\tau/dt$. The neutron's vanishing charge is then a *composite* zero (a sum of quark charges), the neutrino's a *reflected elementary* zero, and a mode in which the two channels average has vanishing effective decay rate—consistent with the saddle ($\operatorname{Re}\Gamma = 0$, free fall rather than exponential decay) that already appears in the τ-dynamics. Two gaps are explicit: (i) the neutron–neutrino torus, if admissible, is a *distinct* object on the fermionic level and must not be conflated with the photonic torus $\mathcal{E}_\tau$ of Sec. II; (ii) the timescale hierarchy (a free neutron at $\sim 10^3\,$s versus cosmological times) is unresolved and is not papered over. Until both are closed this remains a heuristic.

C. Maxwell's equations as a real projection

If the photon is the holomorphic object $\mathcal{E}_\tau$ (Sec. II), the natural conjecture is that the sinusoidal electromagnetic wave is the real shadow of the circular phase on one S^1 factor. This conjecture needs its own real-projection map, distinct from the $\operatorname{Stab}(i)$ observer projection of Sec. II.4: that projection is derived along the canonical trajectory $\operatorname{Re}\tau = 0$ specifically, and Sec. II.4 explicitly retires the naive statement "the observer measures $\operatorname{Re}\tau$" as ill-defined away from it. A generic photon state is not confined to that trajectory, so a $\mathrm{PSL}(2,\mathbb{Z})$-covariant projection valid off it does not yet exist; constructing one is a prerequisite for this subsection, not a detail of it. Granting such a map for the sake of the conjecture, the stronger claim—that the projected field strength F satisfies $dF = 0$ and $d{\star}F = J$ as the consistency condition of the real projection of the modular-invariant action—is a *target theorem, not a result*: it requires (i) constructing the generalized real projection above, (ii) building $F_{\mu\nu}$ from a real-projected holomorphic form on $\mathcal{E}_\tau$, (iii) deriving the Bianchi identity from closedness, (iv) obtaining $d{\star}F = J$ from the projection's consistency condition, and (v) identifying the photon $U(1)$ with one torus circle. If completed it would strengthen the photonic identification (dynamics, not merely topology) without touching the parameter-free chain; if it fails, the photon remains only topologically identified and the conjecture is excluded.

VIII. DISCUSSION

A single meta-postulate—structural realism applied to the concept of time—leads through a chain of necessary steps to a unique, parameter-free prediction, $r = 2(1-n_s)^2 \approx 0.0025$. The framework requires no anthropic reasoning, no parameter tuning, and no appeal to specific string compactifications: $\alpha = 2/3$ is the unique topological invariant of the modular curve that governs the dynamics of complex time. The speculative extensions of Sec. VII are offered separately and conditionally; the falsifiable content of the theory is Eq. (8) and the LiteBIRD/CMB-S4 discrimination of Fig. 1.

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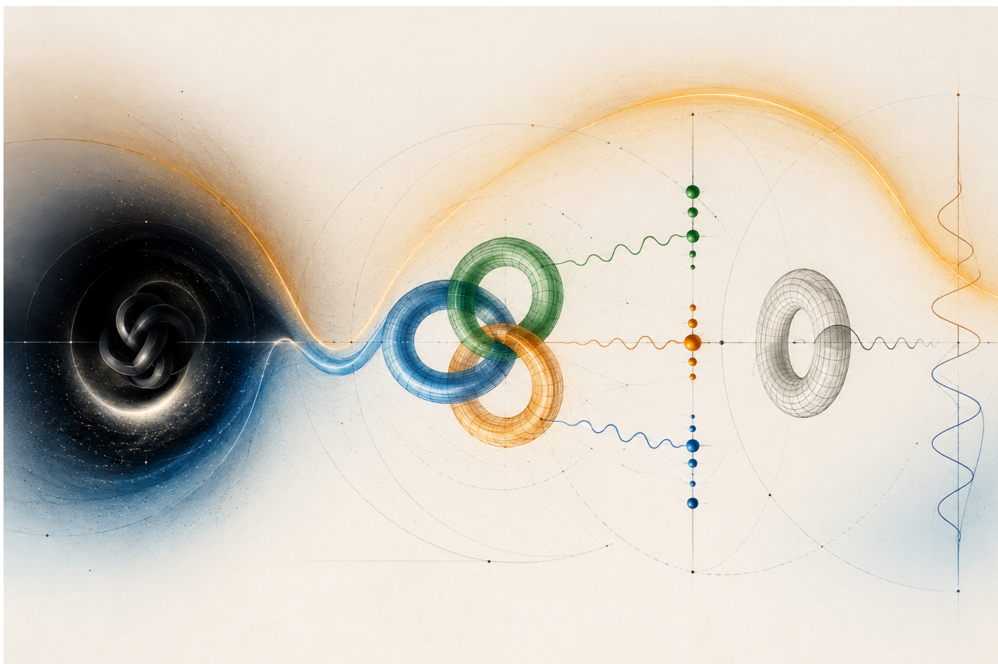

FIG. 2. Conceptual illustration (artistic, non-technical) of the Sec. VII picture: a $2i$ collapse on the left, the interlinked $S^1 \times S^1$ tori, their sinusoidal $\mathrm{Re}\,\tau$ projections, and a discrete generational spectrum. Included for intuition only; it carries no quantitative content.

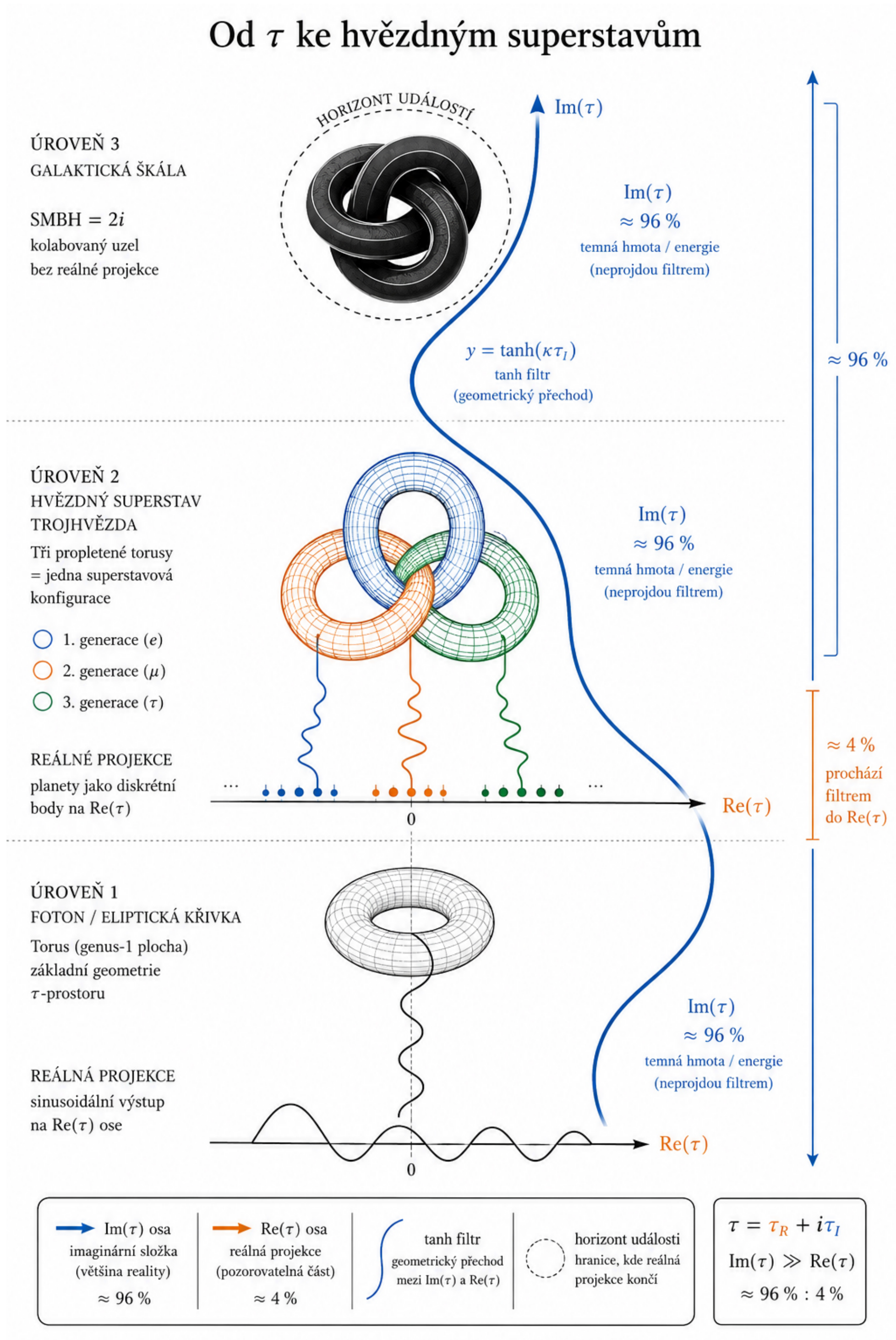

FIG. 3. Conceptual schematic of the three-level τ hierarchy (labels in Czech in the original artwork). *Level 1:* the photon as a genus-1 torus (elliptic curve), whose $\mathrm{Re}\,\tau$ projection is a sinusoidal wave. *Level 2:* a stellar superstate ("three interlinked tori") with planets as discrete points on the $\mathrm{Re}\,\tau$ axis and a generational reading $e/\mu/\tau$. *Level 3:* a supermassive black hole as a collapsed $2i$ node with no real projection (event horizon). The blue axis is $\mathrm{Im}\,\tau$ (~96%, unprojected); the orange axis is $\mathrm{Re}\,\tau$ (~4%, observable); the tanh filter mediates the transition. This figure illustrates the conjectures of Sec. VII only.

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