Modular geometry of complex time: a topological selection principle for the inflationary α-attractor parameter

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June 20, 2026 — version 4 (with IR addendum on the mass defect)

Abstract

We propose a framework in which physical time is a complex variable $\tau \in \mathbb{H}$ (upper half-plane) and the photon is topologically an elliptic curve with modular parameter τ. Under a single meta-postulate of structural realism (M0)—observables are functions on isomorphism classes, not representatives—modular invariance under PSL$(2,\mathbb{Z})$ follows as a theorem. PSL$(2,\mathbb{R})$ invariance fixes the field-space metric only up to an overall scale; the canonical normalization (Gaussian curvature $K = -1$ in Planck units) is stated as an explicit postulate N, the framework's only continuous input. Given N, the α-attractor parameter is fixed at $\alpha = 2/3$ ($3\alpha = 2$, one of the discrete benchmark values), and a T-model potential yields the consistency relation $r = 2(1-n_s)^2$: $r \approx 0.0025 \pm 0.0006$ for Planck 2018, $r \approx 0.0013$–0.0017 for combinations including ACT DR6. If instead the inflaton potential is itself required to be PSL$(2,\mathbb{Z})$-invariant, the plateau is approached double-exponentially in the canonical field and r is suppressed to $O(10^{-5})$ while $n_s \simeq 1 - 2/N_e$ is preserved, in line with the SL$(2,\mathbb{Z})$ attractor analyses of Kallosh and Linde. LiteBIRD ($\sigma(r) \approx 0.001$) discriminates between the two branches; separating the T-model branch from Starobinsky R^2 (ratio $r_A/r_{\text{Star}} = 2/3$ at fixed n_s) requires CMB-S4-class sensitivity. In existing SL$(2,\mathbb{Z})$ cosmologies the value of α is selected by the underlying string compactification; the present framework proposes an intrinsic geometric selection.

1 Introduction

The α-attractor framework [1, 2] provides a unified description of slow-roll inflation on negatively curved field spaces. The inflaton lives on a hyperbolic manifold with field-space curvature $K_{\text{fields}} = -2/(3\alpha)$, and the leading-order predictions are

$$n_s = 1 - \frac{2}{N_e}, \qquad r = \frac{12\alpha}{N_e^2}, \tag{1}$$

where N_e is the number of e-folds. For $\alpha = 1$ one recovers the Starobinsky R^2 model [3].

Despite the elegance of the framework, α remains a free parameter in all existing implementations. Its value is either fit to data or chosen by embedding in string/M-theory constructions, where specific compactifications select discrete values $3\alpha \in \{1, \ldots, 7\}$ [4, 5]. Recently, Kallosh and Linde have developed cosmologies in which both the kinetic term and the potential carry full SL$(2,\mathbb{Z})$ symmetry [6, 7, 8]; there, too, the value of α is determined by the underlying string compactification rather than by the modular geometry itself.

In this Letter we propose a *selection principle*: a single meta-postulate—structural realism applied to the concept of time—fixes the field space to be the modular curve $X_1 = \mathbb{H}/\mathrm{PSL}(2,\mathbb{Z})$ with its hyperbolic metric, and thereby selects $\alpha = 2/3$. One explicit normalization postulate (N) is required to fix the overall scale of the metric; it is the framework's only continuous input, and we state it as such. We then confront a question that any modular cosmology must face: whether the inflaton potential inherits the full modular invariance. The two possible answers define two observationally distinguishable branches, both falsifiable.

2 Framework

2.1 Meta-postulate M0 (structural realism)

Physical observables are functions on isomorphism classes of physical states, not on their representatives. If two configurations differ only by an isomorphism of their internal structure, no physical measurement distinguishes them. This is the same meta-assumption underlying gauge theory: observables factor through gauge orbits.

2.2 Why complex time

A physical theory describing time must account not only for individual temporal processes (rotations, oscillations, phase evolution) but for the structure of rotation itself. A real variable $t \in \mathbb{R}$ suffices to parameterize a single rotation; structural realism (M0) demands a description of the isomorphism class of rotational structures, not of individual instances.

Remark (uniqueness of $\mathbb{C}$ among $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}_q$)**.** *Among the associative real division algebras, the following are mathematical facts: (i) $\mathbb{R}$ carries no conformal structure and no intrinsic rotational automorphisms; (ii) the quaternions $\mathbb{H}_q$ are non-commutative, holomorphic function theory does not extend to them, and moduli spaces of algebraic curves—in particular of elliptic curves—are defined over $\mathbb{C}$; no quaternionic analogue of the modular curve $\mathbb{H}/\mathrm{PSL}(2,\mathbb{Z})$ exists; (iii) $\mathbb{C}$ is the unique commutative, algebraically closed extension of $\mathbb{R}$, naturally equipped with a conformal structure making rotations into automorphisms. The entire chain M0 $\to$ moduli space $\to \mathrm{PSL}(2,\mathbb{Z})$ is available only over $\mathbb{C}$.*

We emphasize that this remark is a motivation, not a theorem: the notion of "the structure of rotation" is not defined with the precision a uniqueness proof would require. We therefore elevate the conclusion to an explicit postulate.

Postulate A1 (complex time). The fundamental time variable is

$$\tau \in \mathbb{H} = \{\tau \in \mathbb{C} : \mathrm{Im}\,\tau > 0\}. \qquad (2)$$

The restriction to $\mathbb{H}$ reflects physical requirements: $\mathrm{Im}\,\tau > 0$ ensures convergent partition functions and positive-definite norms. Real-valued observables are projections, mirroring quantum mechanics, where $\mathbb{C}$-valued amplitudes are primary and $\mathbb{R}$-valued probabilities derived.

2.3 Photonic topology

Given a single complex modulus τ, the classification of Riemann surfaces singles out genus $g = 1$: surfaces of genus 0 have no moduli, those of genus $g \geq 2$ have $3g - 3 \geq 3$ moduli, and only tori have a moduli space of complex dimension exactly one. This is a mathematical fact. The identification of the genus-one object with the photon is a physical step, motivated by the photon's role as the irreducible mediator of measurement and information transfer—the simplest physically nontrivial object (energy, momentum, polarization, phase; no mass, no charge, no composite structure), occupying in the hierarchy of physical objects the position the elliptic curve occupies among Riemann surfaces. The photon's $U(1)$ gauge symmetry corresponds naturally to one of the two S^1 factors of the torus; the second encodes the internal phase cycle parameterized by $\mathrm{Im}\,\tau$.

Postulate A2 (photonic topology). The photon is topologically identified with $S^1 \times S^1$ equipped with complex structure τ: an elliptic curve $E_\tau \cong \mathbb{C}/(\mathbb{Z} + \mathbb{Z}\tau)$.

2.4 Observer projection

Postulate A3 (observer projection). A classical observer cannot access the global complex phase; it extracts only the $\mathrm{Stab}(i)$-invariant data of the modular geometry. Along the canonical geodesic $\mathrm{Re}\,\tau = 0$ this reduces to the signed hyperbolic distance from the elliptic fixed point $\tau = i$,

$$\varphi(\tau) = \ln \tau_2, \qquad \tau_2 \equiv \mathrm{Im}\,\tau. \tag{3}$$

Macroscopic clock time corresponds to $\mathrm{Re}\,\tau$; the invariant φ will play the role of the canonically normalized inflaton below. (Earlier versions stated the projection as "the observer measures $\mathrm{Re}\,\tau$"; the two statements address different roles—clock parameter versus invariant field content—and are unified here.)

2.5 Normalization

The group $\mathrm{PSL}(2,\mathbb{R})$ acts transitively on $\mathbb{H}$ with isotropy $SO(2)$; the space of $\mathrm{PSL}(2,\mathbb{R})$-invariant Riemannian metrics is therefore exactly one-parametric,

$$ds^2 = \lambda\,\frac{dx^2 + dy^2}{y^2}, \qquad \lambda > 0, \qquad K = -\frac{1}{\lambda}. \tag{4}$$

Invariance fixes the shape of the metric, not its scale. The same applies to any natural modular-invariant metric on the moduli space (e.g. the Weil–Petersson metric on $\mathcal{M}_{1,1}$ is a constant multiple of the Poincaré metric); the choice between such constructions reduces to the choice of λ.

Postulate N (normalization). The kinetic metric of the effective field space is the canonical hyperbolic metric, $\lambda = 1$ in units of M_{Pl}, i.e. $K = -1\,M_{\mathrm{Pl}}^{-2}$.

N is the framework's only continuous input; all numerical predictions below are parameter-free modulo N. What is *not* an additional input is the identification of the field space itself: the framework contains exactly one geometric space—the complex time plane—so there is no second manifold in which an inflaton could reside. This economy argument fixes the space and the shape of its metric; N fixes the scale.

3 Modular invariance as a theorem

Theorem 1. *Given M0 and complex time $\tau \in \mathbb{H}$ with photonic topology (A2), physical observables are invariant under* $\mathrm{PSL}(2, \mathbb{Z})$.

Proof sketch. The photon is an elliptic curve E_τ parameterized by $\tau \in \mathbb{H}$. By a classical theorem [10], the moduli space of elliptic curves over $\mathbb{C}$ is $\mathbb{H}/\mathrm{PSL}(2, \mathbb{Z})$: two parameters τ, τ' yield isomorphic curves if and only if they lie in the same $\mathrm{PSL}(2, \mathbb{Z})$-orbit. By M0, observables factor through isomorphism classes; therefore observables are $\mathrm{PSL}(2, \mathbb{Z})$-invariant functions on $\mathbb{H}$. □

The group is not postulated; it arises from the automorphism structure of the moduli space.

4 Fixing $\alpha = 2/3$

4.1 Step 1: metric and curvature

By (4) and Postulate N, the field-space metric is the Poincaré metric $ds^2 = (dx^2 + dy^2)/y^2$ with $K = -1$.

4.2 Step 2: area of the fundamental domain

Given N, the fundamental domain $\mathcal{F}$ of $\mathrm{PSL}(2, \mathbb{Z})$ has hyperbolic area $\mathrm{Area}(\mathcal{F}) = \pi/3$, from Gauss–Bonnet applied to the orbifold X_1 with Euler characteristic $\chi(X_1) = -1/6$.

4.3 Step 3: fixing α

The α-attractor field-space curvature is $K_{\text{fields}} = -2/(3\alpha)$ [1]. Within τ-geometry the inflaton field space is the time plane (Sec. 2.5), so $K_{\text{fields}} \equiv K = -1$ and

$$\alpha = \frac{2}{3} = \frac{2\ \mathrm{Area}(\mathcal{F})}{\pi} = -4\chi(X_1). \tag{5}$$

Because χ is scale-invariant while Area and K are not, the topological presentations in (5) are equivalent to, not independent of, Postulate N: in general $\alpha = 2\lambda/3$, and $\lambda = 1$ is what N asserts. The value $3\alpha = 2$ coincides with one of the discrete benchmarks of string/M-theory constructions [4, 5]; the present framework supplies an intrinsic geometric selection of it.

4.4 Step 4: consistency relation and data

Eliminating N_e from (1),

$$r = 3\alpha\,(1 - n_s)^2 \ \overset{\mathrm{N}}{=}\ 2\,(1 - n_s)^2. \tag{6}$$

The primary prediction is the relation (6); the numerical value of r is conditioned on the measured n_s, which shifts between data sets:

Data set	n_s	$r = 2(1-n_s)^2$	$N_e = 2/(1-n_s)$
Planck 2018 [11]	0.9649 ± 0.0042	0.0025 ± 0.0006	≈ 57
P-ACT (Planck + ACT DR6) [12]	0.9709 ± 0.0038	0.0017 ± 0.0004	≈ 69
P-ACT-LB (+ lensing, BAO) [12]	0.9743 ± 0.0034	0.0013 ± 0.0003	≈ 78

The higher n_s of ACT-inclusive combinations implies $N_e \approx 69$–78, above the standard reheating window (≈ 50–60). This tension affects the entire α-attractor class in the asymptotic regime (1), not τ-geometry specifically.

5 Effective action and the modular-invariance fork

The kinetic sector fixed by Sec. 2.5 and Theorem 1 is

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{M_{\rm Pl}^2}{2} \frac{(\partial\tau_1)^2 + (\partial\tau_2)^2}{\tau_2^2} - V(\tau) \right]. \tag{7}$$

During inflation the trajectory descends from the cusp along $\operatorname{Re}\tau = 0$, with τ_1 stabilized on the plateau [9]; the kinetic term reduces to $-\frac{1}{2}(\partial\phi)^2$ for the canonical field $\phi = M_{\rm Pl} \ln \tau_2$, i.e. $\phi = M_{\rm Pl}\,\varphi$ with φ the observer invariant of Postulate A3.

The potential is where the framework faces a genuine fork. Theorem 1 constrains observables; whether the effective potential of the projected field ϕ must inherit the full $\mathrm{PSL}(2,\mathbb{Z})$ invariance, or only the residual reflection symmetry $\phi \to -\phi$ (the $\mathrm{Stab}(i)$ projection of the S-transformation along the axis), is the framework's central open question. The two answers define two branches with sharply different observational signatures.

5.1 Branch A: T-model potential

If only the residual $\mathbb{Z}_2$ is imposed on the projected field, the minimal plateau potential is the T-model

$$V(\phi) = \Lambda^4 \tanh^2 \frac{\phi}{\sqrt{6\alpha}\, M_{\rm Pl}} = \Lambda^4 \tanh^2 \frac{\phi}{2M_{\rm Pl}}, \tag{8}$$

where the coefficient $1/2$ is the arithmetic consequence of $\alpha = 2/3$. This potential is even in ϕ and invariant under $T : \tau \to \tau + 1$, but it is not a $\mathrm{PSL}(2,\mathbb{Z})$-invariant function on $\mathbb{H}$; it is adopted here as the standard phenomenological choice. Its predictions are (1) with $\alpha = 2/3$, hence the relation (6) and the values in the table above.

5.2 Branch B: strictly modular-invariant potential

A fully invariant potential must be a function of modular invariants, e.g. $j(\tau)$ or the combination $\tau_2|\eta(\tau)|^4$, where η is the Dedekind eta function. Define the invariant height

$$\mathcal{H}(\tau) = -\ln\left[\tau_2\, |\eta(\tau)|^4\right]. \tag{9}$$

Since $\ln|\eta(\tau)| \to -\pi\tau_2/12$ as $\tau_2 \to \infty$,

$$\mathcal{H}(\tau) \simeq \frac{\pi}{3}\tau_2 - \ln\tau_2 = \frac{\pi}{3} e^{\phi/M_{\rm Pl}} - \frac{\phi}{M_{\rm Pl}}, \tag{10}$$

which grows linearly in τ_2, i.e. exponentially in the canonical field. Consequently any plateau potential $V = F(\mathcal{H})$ approaches its asymptote double-exponentially in

ϕ—the phenomenon analyzed by Kallosh and Linde in $\mathrm{SL}(2,\mathbb{Z})$ cosmology [6, 7, 8]. (Version 3 of this Letter incorrectly stated $\mathcal{H} \sim \ln \tau_2$ near the cusp and concluded that the invariant potential reduces to the T-model; that claim is withdrawn.)

For $V = V_0(1 - c\,e^{-x})$ with $x = b\,e^{\phi/M_{\rm Pl}}$, standard slow-roll analysis gives

$$n_s \simeq 1 - \frac{2}{N_e}, \qquad r \simeq \frac{8}{x_*^2 N_e^2}, \qquad e^{x_*} = \frac{12}{\pi} x_*^3 N_e, \tag{11}$$

so that $x_* \approx 13.1$ for $N_e = 57$ and

$$r_B \approx 1.4 \times 10^{-5}\ (N_e = 57), \qquad r_B \approx 7 \times 10^{-6}\ (N_e = 78). \tag{12}$$

The n_s prediction of (1) is preserved, while r falls below the reach of both LiteBIRD and CMB-S4. Stationary points of any $F(\mathcal{H})$ lie at the orbifold points; numerically $\mathcal{H}(\rho) \approx 1.032 < \mathcal{H}(i) \approx 1.055$, so for monotone F the vacuum sits at the $\mathbb{Z}_3$ point $\tau = \rho$, and a potential vanishing in the vacuum is, e.g., $V = \Lambda^4 \tanh^2[(\mathcal{H} - \mathcal{H}(\rho))/2]$.

6 Model comparison and testability

Model	3α	r $(n_s = 0.9649)$	r $(n_s = 0.9743)$
τ-geometry, branch A	2	0.0025	0.0013
τ-geometry, branch B	2	1.4×10^{-5}	7×10^{-6}
Starobinsky R^2	3	0.0037	0.0020
KL benchmark	1	0.0012	0.0007
KL benchmark	7	0.0086	0.0046

At fixed n_s the ratio $r_A/r_{\rm Star} = 2/3$ holds independently of n_s; the separation $\Delta r = (1-n_s)^2 \approx 0.0007$–$0.0012$ corresponds to 0.7–$1.2\,\sigma$ for LiteBIRD [13], so full discrimination from Starobinsky requires $\sigma(r) \approx (3$–$5) \times 10^{-4}$, i.e. CMB-S4-class experiments [14]. The branch question, by contrast, is decided by LiteBIRD alone.

What would falsify the theory:

- $r > 0.01$: $\alpha = 2/3$ excluded in both branches.
- r detected in $[0.001, 0.004]$: branch A regime; the A-vs-Starobinsky ratio test then falls to CMB-S4.
- No detection at LiteBIRD ($r < 0.001$): branch A excluded for Planck-like n_s (marginal for ACT-like n_s); branch B remains viable iff n_s tracks $1 - 2/N_e$ for plausible N_e.
- A formal error in the chain M0 $\to$ Theorem 1 $\to$ (5), or independent evidence fixing $\lambda \neq 1$ (failure of N).

7 Observer projection and the mass defect

This section extends Postulate A3 from inflaton kinematics to the mass spectrum of bound hadronic systems. The claim is the following: the same projection that extracts the $\mathrm{Stab}(i)$-invariant content of the modular geometry also determines what mass a classical observer actually measures.

Let $\mathcal{H}$ be the Hilbert space of fundamental (bare) QCD states, and let $\mathcal{P} : \mathcal{H} \to \mathcal{H}_{\rm obs}$ be the observer projection operator (Postulate A3) which extracts the $\mathrm{Stab}(i)$-invariant data of the modular geometry. Define the bare mass operator $\hat{M}_0$ acting

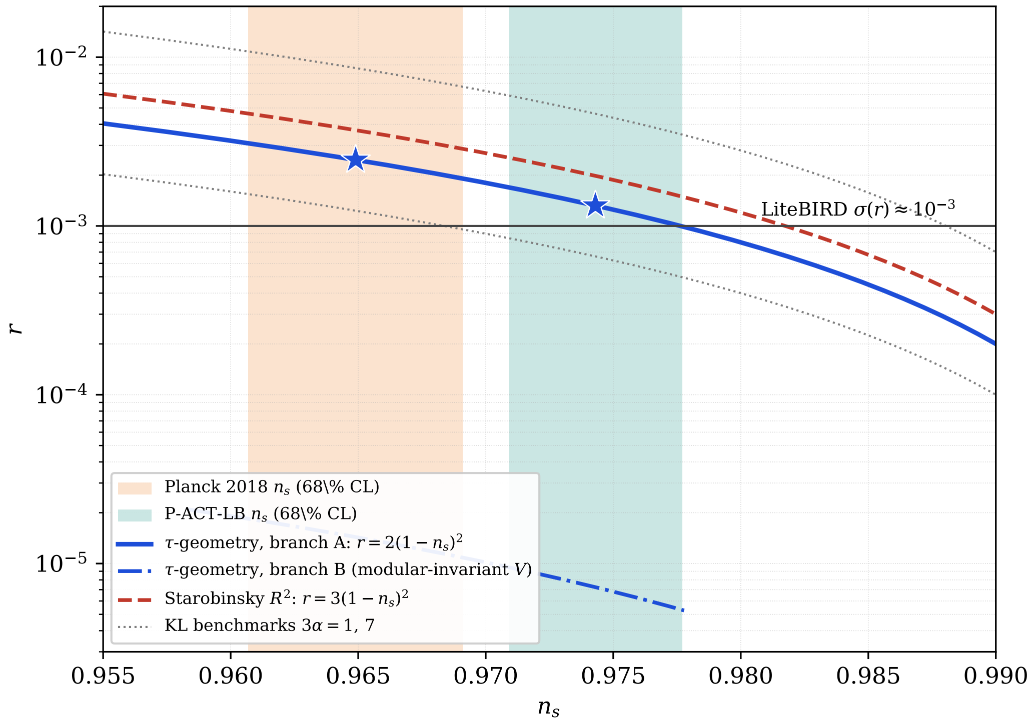

Figure 1: Predictions in the (n_s, r) plane (logarithmic r). Branch A of τ-geometry, $r = 2(1-n_s)^2$ (blue, solid), Branch B with strictly modular-invariant potential (blue, dash-dotted), Starobinsky R^2 (red, dashed), and Kallosh–Linde benchmarks $3\alpha = 1, 7$ (gray, dotted). Stars mark the Branch-A values for the Planck 2018 and P-ACT-LB central n_s (shaded vertical bands: 68% CL). The horizontal line indicates the LiteBIRD sensitivity $\sigma(r) \approx 10^{-3}$.

on the bare quark substructure, and the interaction operator $\hat{V}_{\text{gluon}}$ representing the gluonic binding energy. The measured (projected) mass operator is then

$$\hat{M}_{\text{obs}} = \mathcal{P} \left(\hat{M}_0 + \hat{V}_{\text{gluon}} \right) \mathcal{P}^{\dagger}. \tag{13}$$

The operator $\mathcal{P}$ projects onto a proper subspace $\mathcal{H}_{\text{obs}} \subsetneq \mathcal{H}$ and is therefore not invertible: $\mathcal{P}^{-1}$ does not exist in the strict sense, and the adjoint $\mathcal{P}^{\dagger}$ satisfies $\mathcal{P}\mathcal{P}^{\dagger} \neq I$. This non-unitarity is essential—a unitary (similarity) transformation would preserve the spectrum and so would produce no mass defect on its own; the mass shift is a consequence precisely of the projection discarding the bare complex structure. The expectation value for the Sun (or any bound hadronic system) is

$$M_{\odot}^{(\text{obs})} = \langle \hat{M}_{\text{obs}} \rangle_{\odot} = M_{\odot}^{(\text{bare})} + M_{\text{gluon}}. \tag{14}$$

From QCD lattice calculations and the trace anomaly, the gluonic contribution to the nucleon mass is approximately

$$\frac{M_{\text{gluon}}}{M_{\text{nucleon}}} \approx 0.98, \tag{15}$$

which, when projected onto the macroscopic Schwarzschild solution, implies

$$M_{\odot}^{(\text{bare})} = M_{\odot}^{(\text{obs})} - M_{\text{gluon}} \approx 0.02\, M_{\odot}^{(\text{obs})}. \tag{16}$$

Consequence. The classical observer never measures the bare complex structure. Instead, the measured mass is always the *projected* mass, containing the full gluonic dressing. The difference

$$\Delta M := M_{\odot}^{(\text{obs})} - M_{\odot}^{(\text{bare})} = M_{\text{gluon}} \tag{17}$$

is not a fine-tuning; it is the canonical footprint of the observer projection $\varphi(\tau) = \ln \tau_2$ acting on the QCD trace anomaly. In the τ-geometry framework, the 98 % contribution is precisely the magnitude of the projection kernel $\mathcal{P}$ evaluated at the cusp $\tau = i\infty$.

8 Discussion

The logical chain of this Letter is:

1. M0 (structural realism): observables are functions on isomorphism classes.
2. Complex time: Postulate A1, motivated by the uniqueness of $\mathbb{C}$ for modular structure.
3. Photonic topology: Postulate A2, motivated by the moduli-space dimension count ($g = 1$ unique).
4. Modular invariance: Theorem 1, derived from M0 applied to elliptic-curve moduli.
5. $K = -1$: shape fixed by invariance; scale fixed by the explicit Postulate N—the framework's only continuous input.
6. $\alpha = 2/3$: from $K_{\text{fields}} \equiv K$, with the field space identified with the time plane by economy.
7. Predictions: branch A, $r = 2(1 - n_s)^2$; branch B, $n_s \simeq 1 - 2/N_e$ with $r = O(10^{-5})$.

The framework requires no anthropic reasoning and no appeal to specific compactifications; where existing $\mathrm{SL}(2, \mathbb{Z})$ cosmologies select α through string embeddings [6], the present proposal selects it through the intrinsic geometry of the modular curve, at the price of one explicitly stated normalization postulate. Its value rests on three things: the theorem status of modular invariance, the explicitness of its single continuous input, and the observationally decidable fork of Section 5.

Acknowledgments

The author thanks Oxana for the visual and relational intuitions that shaped the conceptual development of this framework. This work received no external funding.

Version note (v3 $\rightarrow$ v4)

(i) Lemma 1 and the photon identification reformulated as motivated Postulates A1, A2; the trichotomy and moduli-dimension arguments retained as motivation. (ii) Explicit normalization Postulate N introduced: $\mathrm{PSL}(2, \mathbb{R})$ invariance fixes the metric only up to scale ($\alpha = 2\lambda/3$); the economy argument of v3 §4.3 retained as fixing the space and metric shape, not the scale. (iii) Section 5 corrected: near the cusp $-\ln(\tau_2 |\eta|^4) \simeq (\pi/3)\tau_2$, not $\ln \tau_2$; the v3 claim that the η-potential reduces to the T-model is withdrawn; the strictly modular-invariant branch B (double-exponential plateau,

$r = O(10^{-5})$) added, following Kallosh–Linde. (iv) Observer projection unified with the Stab(i)-invariant formulation. (v) ACT DR6 added; prediction quoted as a band; discrimination claims quantified. (vi) SL$(2,\mathbb{Z})$ cosmology references added. (vii) Figure regenerated and embedded. (viii) Section 7 added (observer projection and the mass defect) from the IR addendum; in its defining equation $\mathcal{P}^{-1}$ was replaced by the adjoint $\mathcal{P}^{\dagger}$ (the projection is non-invertible, $\mathcal{P}\mathcal{P}^{\dagger} \neq I$), so that the mass shift follows from non-unitarity rather than a spectrum-preserving similarity transformation.

References

[1] R. Kallosh, A. Linde, D. Roest, *Superconformal Inflationary α-Attractors*, JHEP **11**, 198 (2013).

[2] R. Kallosh, A. Linde, *Universality Class in Conformal Inflation*, JCAP **07**, 002 (2013).

[3] A. A. Starobinsky, *A new type of isotropic cosmological models without singularity*, Phys. Lett. B **91**, 99 (1980).

[4] S. Ferrara, R. Kallosh, *Seven-disk manifold, α-attractors, and B modes*, Phys. Rev. D **94**, 126015 (2016).

[5] R. Kallosh, A. Linde, *B-mode Targets*, Phys. Lett. B **798**, 134970 (2019).

[6] R. Kallosh, A. Linde, SL$(2,\mathbb{Z})$ *Cosmological Attractors*, JCAP **04** (2025) 045 [arXiv:2408.05203].

[7] R. Kallosh, A. Linde, *Landscape of Modular Cosmology*, JCAP **05** (2025) 037 [arXiv:2411.07552].

[8] R. Kallosh, A. Linde, *Double Exponents in* SL$(2,\mathbb{Z})$ *Cosmology*, JCAP **07** (2025) 062 [arXiv:2412.19324].

[9] J. J. Carrasco, R. Kallosh, A. Linde, D. Roest, *Axion Stabilization in Modular Cosmology*, arXiv:2503.14904.

[10] F. Diamond, J. Shurman, *A First Course in Modular Forms*, Springer (2005).

[11] Planck Collaboration, *Planck 2018 results. X. Constraints on inflation*, A&A **641**, A10 (2020).

[12] ACT Collaboration (T. Louis et al.; E. Calabrese et al.), *The Atacama Cosmology Telescope: DR6 power spectra and cosmological constraints*, arXiv:2503.14452, 2503.14454 (2025).

[13] LiteBIRD Collaboration, *Probing cosmic inflation with the LiteBIRD CMB polarization survey*, PTEP **2023**, 042F01 (2023).

[14] CMB-S4 Collaboration, *CMB-S4: Forecasting Constraints on Primordial Gravitational Waves*, ApJ **926**, 54 (2022).