Modular geometry of complex time: a topological selection principle for the inflationary α-attractor parameter

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(SPT-3G D1 data; post-CMB-S4 landscape; epistemological-robustness subsection in Discussion)

Abstract

We propose a framework in which physical time is a complex variable $\tau \in \mathbb{H}$ (upper half-plane) and the photon is topologically an elliptic curve with modular parameter τ. Under a single meta-postulate of structural realism (M0)—observables are functions on isomorphism classes, not representatives—modular invariance under $PSL(2,\mathbb{Z})$ follows as a theorem. $PSL(2,\mathbb{R})$ invariance fixes the field-space metric only up to an overall scale; the canonical normalization (Gaussian curvature $K = -1$ in Planck units) is stated as an explicit postulate N, the framework's only continuous input. A further explicit postulate (A4, new in this version) identifies the inflaton field space with the modular time plane itself; we flag this as the framework's least independently motivated assumption. Given N and A4, the α-attractor parameter is fixed at $\alpha = 2/3$ ($3\alpha = 2$, one of the discrete benchmark values), and a T-model potential yields the consistency relation $r = 2(1-n_s)^2$: $r \approx 0.0025 \pm 0.0006$ for Planck 2018, $r \approx 0.0020 \pm 0.0004$ for the joint SPT+Planck+ACT data set (CMB-SPA), and $r \approx 0.0013$–0.0017 for combinations including BAO. If instead the inflaton potential is itself required to be $PSL(2,\mathbb{Z})$-invariant, the plateau is approached double-exponentially in the canonical field and r is suppressed to $O(10^{-5})$ while $n_s \simeq 1 - 2/N_e$ is preserved, in line with the $SL(2,\mathbb{Z})$ attractor analyses of Kallosh and Linde. LiteBIRD ($\sigma(r) \approx 0.001$; launch now targeted for JFY2036) remains the experiment that discriminates between the two branches; in the nearer term, Simons Observatory ($\sigma(r) \approx 0.003$) begins to probe the upper part of the branch-A band. Separating the T-model branch from Starobinsky R^2 (ratio $r_A/r_{\text{Star}} = 2/3$ at fixed n_s) requires sensitivity $\sigma(r) \approx (3$–$5) \times 10^{-4}$, a target left without a funded carrier by the July 2025 cancellation of CMB-S4. Section 7 reinterprets, rather than derives, the QCD nucleon mass defect via the observer projection $\mathcal{P}$; it makes no new numerical prediction. A speculative entropic-gravity outlook is added in Section 8.1.

Contents

1 Introduction

The α-attractor framework [1, 2] provides a unified description of slow-roll inflation on negatively curved field spaces. The inflaton lives on a hyperbolic manifold with field-space curvature $K_{\text{fields}} = -2/(3\alpha)$, and the leading-order predictions are

$$n_s = 1 - \frac{2}{N_e}, \qquad r = \frac{12\alpha}{N_e^2}, \tag{1}$$

where N_e is the number of e-folds. For $\alpha = 1$ one recovers the Starobinsky R^2 model [3].

Despite the elegance of the framework, α remains a free parameter in all existing implementations. Its value is either fit to data or chosen by embedding in string/M-theory constructions, where specific compactifications select discrete values $3\alpha \in \{1, \ldots, 7\}$ [4, 5]. Recently, Kallosh and Linde have developed cosmologies in which both the kinetic term and the potential carry full $SL(2, \mathbb{Z})$ symmetry [6, 7, 8]; there, too, the value of α is determined by the underlying string compactification rather than by the modular geometry itself.

In this Letter we propose a selection principle: a single meta-postulate—structural realism applied to the concept of time—fixes the field space to be the modular curve $X_1 = \mathbb{H}/PSL(2, \mathbb{Z})$ with its hyperbolic metric, and thereby selects $\alpha = 2/3$. One explicit normalization postulate (N) is required to fix the overall scale of the metric, and one explicit identification postulate (A4) is required to connect that geometry to the inflaton kinetic term; both are the framework's continuous and structural inputs, and we state them as such. We then confront a question that any modular cosmology must face: whether the inflaton potential inherits the full modular invariance. The two possible answers define two observationally distinguishable branches, both falsifiable.

2 Framework

2.1 Meta-postulate M0 (structural realism)

Physical observables are functions on isomorphism classes of physical states, not on their representatives. If two configurations differ only by an isomorphism of their internal structure, no physical measurement distinguishes them. This is the same meta-assumption underlying gauge theory: observables factor through gauge orbits.

2.2 Why complex time

A physical theory describing time must account not only for individual temporal processes (rotations, oscillations, phase evolution) but for the structure of rotation itself. A real variable $t \in \mathbb{R}$ suffices to parameterize a single rotation; structural realism (M0) demands a description of the isomorphism class of rotational structures, not of individual instances.

Remark (uniqueness of $\mathbb{C}$ among $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}_q$). Among the associative real division algebras, the following are mathematical facts: (i) $\mathbb{R}$ carries no conformal structure and no intrinsic rotational automorphisms; (ii) the quaternions $\mathbb{H}_q$ are non-commutative, holomorphic function theory does not extend to them, and moduli spaces of algebraic curves—in particular of elliptic curves—are defined over $\mathbb{C}$; no quaternionic analogue of the modular curve $\mathbb{H}/PSL(2,\mathbb{Z})$ exists; (iii) $\mathbb{C}$ is the unique commutative, algebraically closed extension of $\mathbb{R}$, naturally equipped with a conformal structure making rotations into automorphisms. The entire chain M0 $\rightarrow$ moduli space $\rightarrow PSL(2,\mathbb{Z})$ is available only over $\mathbb{C}$.

We emphasize that this remark is a motivation, not a theorem: the notion of "the structure of rotation" is not defined with the precision a uniqueness proof would require. We therefore elevate the conclusion to an explicit postulate.

Postulate 1 (A1: complex time)**.** *The fundamental time variable is*

$$\tau \in \mathbb{H} = \{\tau \in \mathbb{C} : \operatorname{Im}\tau > 0\}.$$

The restriction to $\mathbb{H}$ reflects physical requirements: $\operatorname{Im}\tau > 0$ ensures convergent partition functions and positive-definite norms. Real-valued observables are projections, mirroring quantum mechanics, where $\mathbb{C}$-valued amplitudes are primary and $\mathbb{R}$-valued probabilities derived.

2.3 Photonic topology

Given a single complex modulus τ, the classification of Riemann surfaces singles out genus $g = 1$: surfaces of genus 0 have no moduli, those of genus $g \geq 2$ have $3g - 3 \geq 3$ moduli, and only tori have a moduli space of complex dimension exactly one. This is a mathematical fact. The identification of the genus-one object with the photon is a physical step, motivated by the photon's role as the irreducible mediator of measurement and information transfer—the simplest physically non-trivial object (energy, momentum, polarization, phase; no mass, no charge, no composite structure), occupying in the hierarchy of physical objects the position the elliptic curve occupies among Riemann surfaces. The photon's $U(1)$ gauge symmetry corresponds naturally to one of the two S^1 factors of the torus; the second encodes the internal phase cycle parameterized by $\operatorname{Im}\tau$.

Postulate 2 (A2: photonic topology)**.** *The photon is topologically identified with $S^1 \times S^1$ equipped with complex structure τ: an elliptic curve $E_\tau \cong \mathbb{C}/(\mathbb{Z} + \mathbb{Z}\tau)$.*

2.4 Observer projection

Postulate 3 (A3: observer projection)**.** *A classical observer cannot access the global complex phase; it extracts only the* Stab(i)*-invariant data of the modular geometry. Along the canonical geodesic* $\operatorname{Re}\tau = 0$ *this reduces to the signed hyperbolic distance from the elliptic fixed point* $\tau = i$*,*

$$\varphi(\tau) = \ln \tau_2, \qquad \tau_2 \equiv \operatorname{Im}\tau.$$

Macroscopic clock time corresponds to $\operatorname{Re}\tau$; the invariant φ will play the role of the canonically normalized inflaton below.

2.5 Normalization

The group $PSL(2,\mathbb{R})$ acts transitively on $\mathbb{H}$ with isotropy $SO(2)$; the space of $PSL(2,\mathbb{R})$-invariant Riemannian metrics is therefore exactly one-parametric,

$$ds^2 = \lambda\,\frac{dx^2 + dy^2}{y^2}, \qquad \lambda > 0, \qquad K = -\frac{1}{\lambda}. \tag{2}$$

Invariance fixes the shape of the metric, not its scale.

Postulate 4 (N: normalization)**.** *The kinetic metric of the effective field space is the canonical hyperbolic metric,* $\lambda = 1$ *in units of* M_{Pl}*, i.e.* $K = -1\,M_{\mathrm{Pl}}^{-2}$*.*

N is the framework's only continuous input; all numerical predictions below are parameter-free modulo N.

Postulate 5 (A4: field-space identification — **new in v6**)**.** *The inflaton field space of the effective four-dimensional theory is identified with the modular time plane* $\mathbb{H}$ *of Postulate A1, equipped with the metric of Postulate N. Equivalently: the same geometric object that carries the complex-time structure also carries the inflaton kinetic term.*

This is an independent physical postulate, not a consequence of A1–A3 and N. It is the minimal assumption under which the modular-curve geometry of Section 4 has any bearing on inflationary observables at all; without it, τ would remain a purely kinematic time variable with no connection to (n_s, r). We flag it explicitly here because it is where the framework's predictive claim and its ontological claim (complex time) are joined by fiat rather than derivation. Of the framework's postulates, A4 carries the least independent motivation: it is asserted for its predictive economy rather than derived. A future version should either supply an argument for A4 or reformulate the prediction chain without it.

3 Modular invariance as a theorem

Theorem 1. *Given M0 and complex time* $\tau \in \mathbb{H}$ *with photonic topology (A2), physical observables are invariant under* $PSL(2,\mathbb{Z})$*.*

Proof sketch. The photon is an elliptic curve E_τ parameterized by $\tau \in \mathbb{H}$. By a classical theorem [10], the moduli space of elliptic curves over $\mathbb{C}$ is $\mathbb{H}/PSL(2,\mathbb{Z})$: two parameters τ, τ' yield isomorphic curves if and only if they lie in the same $PSL(2,\mathbb{Z})$-orbit. By M0, observables factor through isomorphism classes; therefore observables are $PSL(2,\mathbb{Z})$-invariant functions on $\mathbb{H}$. □

The group is not postulated; it arises from the automorphism structure of the moduli space.

4 Fixing $\alpha = 2/3$

4.1 Step 1: metric and curvature

By (2) and Postulate N, the field-space metric is the Poincaré metric $ds^2 = (dx^2 + dy^2)/y^2$ with $K = -1$.

4.2 Step 2: area of the fundamental domain

Given N, the fundamental domain $\mathcal{F}$ of $PSL(2,\mathbb{Z})$ has hyperbolic area $\mathrm{Area}(\mathcal{F}) = \pi/3$, from Gauss–Bonnet applied to the orbifold X_1 with Euler characteristic $\chi(X_1) = -1/6$.

4.3 Step 3: fixing α

The α-attractor field-space curvature is $K_{\text{fields}} = -2/(3\alpha)$ [1]. By Postulate A4, the inflaton field space is the time plane (Sec. 2.5), so $K_{\text{fields}} \equiv K = -1$ and

$$\alpha = \frac{2}{3} = \frac{2\,\mathrm{Area}(\mathcal{F})}{\pi} = -4\chi(X_1). \tag{3}$$

Because χ is scale-invariant while Area and K are not, the topological presentations in (3) are equivalent to, not independent of, Postulate N: in general $\alpha = 2\lambda/3$, and $\lambda = 1$ is what N asserts. The value $3\alpha = 2$ coincides with one of the discrete benchmarks of string/M-theory constructions [4, 5]; the present framework supplies an intrinsic geometric selection of it, conditional on A4.

4.4 Step 4: consistency relation and data

Eliminating N_e from (1),

$$r = 3\alpha\,(1 - n_s)^2 \xrightarrow{\text{N, A4}} 2\,(1 - n_s)^2. \tag{4}$$

The primary prediction is the relation (4); the numerical value of r is conditioned on the measured n_s:

Data set	n_s	$r = 2(1 - n_s)^2$	$N_e = 2/(1 - n_s)$
Planck 2018 [11]	0.9649 ± 0.0042	0.0025 ± 0.0006	≈ 57
P-ACT (Planck+ACT DR6) [12]	0.9709 ± 0.0038	0.0017 ± 0.0004	≈ 69
CMB-SPA (SPT+Planck+ACT) [13]	0.9684 ± 0.0030	0.0020 ± 0.0004	≈ 63
P-ACT-LB (+lensing, BAO) [12]	0.9743 ± 0.0034	0.0013 ± 0.0003	≈ 78
CMB-SPA + DESI BAO [13, 14]	0.9728 ± 0.0027	0.0015 ± 0.0003	≈ 74

The SPT-3G D1 release [13] shifts the CMB-only central value of n_s *downward* relative to ACT-inclusive combinations: the joint CMB-SPA constraint, $n_s = 0.9684 \pm 0.0030$, lies between Planck 2018 and P-ACT, implying $N_e \approx 63$ — only mildly above the standard reheating window ($\approx$ 50–60). The stronger tension arises when BAO data are added: BAO-inclusive combinations give $n_s \approx$ 0.973–0.974 and $N_e \approx$ 74–78, and disfavor at more than 2σ the n_s values preferred by Planck alone [14]. Two caveats apply. First, this tension affects the entire α-attractor class in the asymptotic regime (1) — including Starobinsky R^2 — not τ-geometry specifically. Second, the upward pull on n_s is driven by the BAO data themselves, which are in 2.8σ tension with the combined CMB experiments within ΛCDM [13]; until that tension is resolved, the CMB-only and BAO-inclusive rows of the table should be read as bracketing the prediction rather than as competing measurements of it.

5 Effective action and the modular-invariance fork

The kinetic sector fixed by Sec. 2.5 and Theorem 1 is

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{M_{\rm Pl}^2}{2}\frac{(\partial\tau_1)^2+(\partial\tau_2)^2}{\tau_2^2} - V(\tau)\right]. \tag{5}$$

During inflation the trajectory descends from the cusp along $\mathrm{Re}\,\tau = 0$, with τ_1 stabilized on the plateau [9]; the kinetic term reduces to $-\frac{1}{2}(\partial\varphi)^2$ for the canonical field $\varphi = M_{\rm Pl}\ln\tau_2$. The potential is where the framework faces a genuine fork.

5.1 Branch A: T-model potential

If only the residual $\mathbb{Z}_2$ is imposed on the projected field, the minimal plateau potential is the T-model

$$V(\varphi) = \Lambda^4\tanh^2\!\left(\frac{\varphi}{\sqrt{6\alpha}\,M_{\rm Pl}}\right) = \Lambda^4\tanh^2\!\left(\frac{\varphi}{2M_{\rm Pl}}\right), \tag{6}$$

where the coefficient 1/2 is the arithmetic consequence of $\alpha = 2/3$. Its predictions are (1) with $\alpha = 2/3$, hence relation (4) and the values in the table above.

5.2 Branch B: strictly modular-invariant potential

A fully invariant potential must be a function of modular invariants, e.g. $j(\tau)$ or the combination $\tau_2|\eta(\tau)|^4$, where η is the Dedekind eta function. Define the invariant height

$$H(\tau) = -\ln\!\big(\tau_2\,|\eta(\tau)|^4\big)\,. \tag{7}$$

Since $\ln|\eta(\tau)| \to -\pi\tau_2/12$ as $\tau_2\to\infty$,

$$H(\tau) \simeq \frac{\pi}{3}\tau_2 - \ln\tau_2 = \frac{\pi}{3}e^{\varphi/M_{\rm Pl}} - \frac{\varphi}{M_{\rm Pl}}, \tag{8}$$

which grows linearly in τ_2, i.e. exponentially in the canonical field. Consequently any plateau potential $V = F(H)$ approaches its asymptote double-exponentially in φ—the phenomenon analyzed by Kallosh and Linde in $SL(2,\mathbb{Z})$ cosmology [6, 7, 8].

For $V = V_0(1 - c\,e^{-x})$ with $x = b\,e^{\varphi/M_{\rm Pl}}$, standard slow-roll analysis gives

$$n_s \simeq 1 - \frac{2}{N_e}, \qquad r \simeq \frac{8}{x_*^2 N_e^2}, \qquad e^{x_*} = \frac{12}{\pi}x_*^3 N_e, \tag{9}$$

so that $x_* \approx 13.1$ for $N_e = 57$ and

$$r_B \approx 1.4\times10^{-5}\ (N_e = 57), \qquad r_B \approx 7\times10^{-6}\ (N_e = 78). \tag{10}$$

The n_s prediction (1) is preserved, while r falls below the reach of both LiteBIRD and CMB-S4.

6 Model comparison and testability

Model	3α	r ($n_s = 0.9649$)	r ($n_s = 0.9684$)	r ($n_s = 0.9743$)
τ-geometry, branch A	2	0.0025	0.0020	0.0013
τ-geometry, branch B	2	1.4×10^{-5}	1.2×10^{-5}	7×10^{-6}
Starobinsky R^2	3	0.0037	0.0030	0.0020
KL benchmark	1	0.0012	0.0010	0.0007
KL benchmark	7	0.0086	0.0070	0.0046

At fixed n_s the ratio $r_A/r_{\rm Star} = 2/3$ holds independently of n_s; the separation $\Delta r = (1-n_s)^2 \approx 0.0007$–$0.0012$ corresponds to 0.7–$1.2\,\sigma$ for LiteBIRD [19], so full discrimination from Starobinsky requires $\sigma(r) \approx (3–5) \times 10^{-4}$, i.e. CMB-S4-class sensitivity [20]. The branch question, by contrast, is decided by LiteBIRD alone.

Experimental landscape (as of mid-2026). The above testability statements must now be read against a substantially changed landscape. On July 9, 2025, DOE and NSF jointly withdrew support for CMB-S4 [17]; the $\sigma(r) \approx (3–5)\times 10^{-4}$ sensitivity needed for the ratio test $r_A/r_{\rm Star} = 2/3$ currently has no funded carrier, and that test becomes a longer-horizon target contingent on a successor program. LiteBIRD, following the withdrawal of KEK from its original hardware role and a one-year reformation study, was cleared by ISAS/JAXA in September 2025 to proceed, with launch now targeted for JFY2036 and a mission definition review planned for mid-2026 [18]; the branch A/B discrimination therefore shifts to the late 2030s. In the interim, the most sensitive probes are the Simons Observatory, already observing, with forecast $\sigma(r) \approx 0.003$ [16], and the BICEP Array; current B-mode limits are $r < 0.036$ (95%, BK18) and $r < 0.25$ from the first SPT-3G B-mode analysis [15]. Simons Observatory can thus begin to cut into the upper part of the branch-A band ($r \approx 0.0025$ for Planck-like n_s) but cannot reach the BAO-inclusive values $r \approx 0.0013$–0.0015. Branch A is therefore falsifiable but, for the lower range of n_s-dependent predictions, not before LiteBIRD.

Falsification criteria:

- $r > 0.01$: $\alpha = 2/3$ excluded in both branches. (Already testable now: Simons Observatory and BICEP Array.)
- r detected in $[0.001, 0.004]$: branch A regime; the A-vs-Starobinsky ratio test then requires a CMB-S4-class successor.
- No detection at LiteBIRD ($r < 0.001$; late 2030s on the current schedule): branch A excluded for Planck-like n_s (marginal for ACT-like n_s); branch B remains viable iff n_s tracks $1-2/N_e$ for plausible N_e.
- A formal error in the chain M0 $\to$ Theorem 1 $\to$ (3), or independent evidence fixing $\lambda \neq 1$ (failure of N), or independent evidence against A4.

7 Observer projection as a reinterpretation of the mass defect

We stress at the outset what this section does and does not claim. The numerical content—the gluonic fraction of the nucleon mass, Eq. (13) below—is the standard QCD trace-anomaly result and is not derived here; it is imported from lattice QCD. What follows is a reinterpretation of that known result in the language of the observer projection $\mathcal{P}$ introduced in Postulate A3. This section makes no new numerical prediction and is not, by itself, falsifiable independently of the QCD input it reinterprets. Its purpose is structural: to test whether $\mathcal{P}$, motivated originally by inflaton kinematics, extends coherently to a second physical context. We regard this as a plausibility check on A3, not as evidence for it.

Let $\mathcal{H}$ be the Hilbert space of fundamental (bare) QCD states, and let $\mathcal{P} : \mathcal{H} \to \mathcal{H}_{\rm obs}$ be the observer projection operator (Postulate A3). Define the bare mass operator $\hat{M}_0$ acting on the bare quark substructure, and the interaction operator $\hat{V}_{\rm gluon}$ representing the gluonic binding energy.

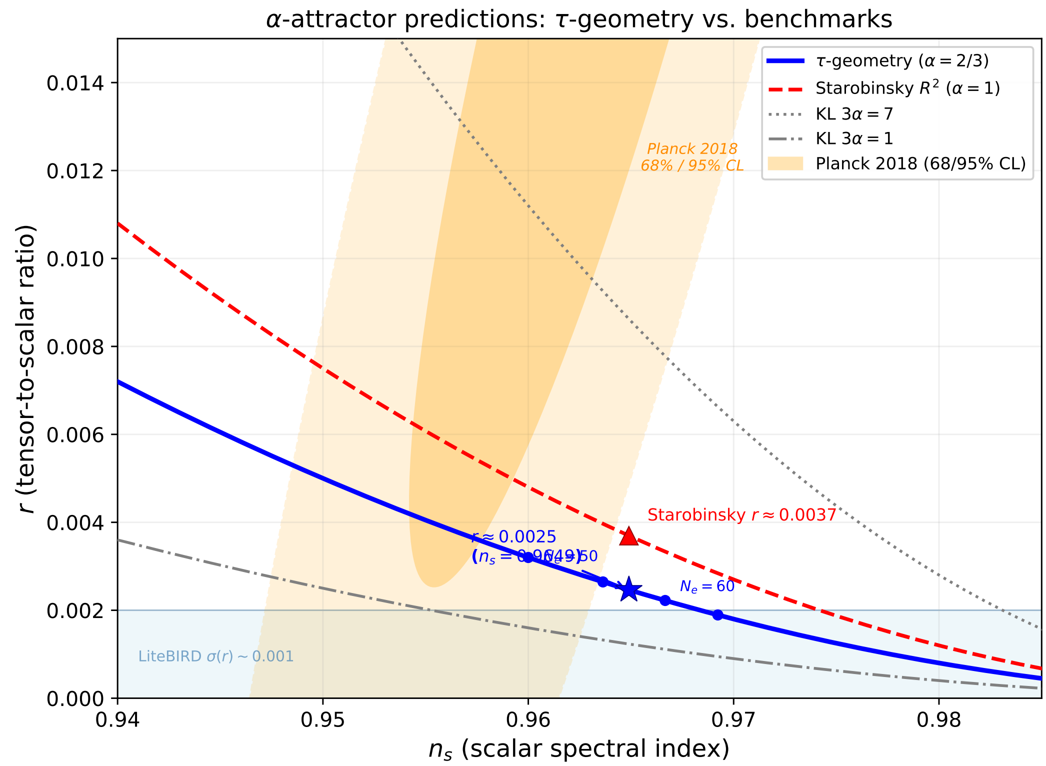

Figure 1: Predictions in the (n_s, r) plane (logarithmic r). Branch A of τ-geometry, $r = 2(1 - n_s)^2$ (blue, solid), Branch B with strictly modular-invariant potential (blue, dash-dotted), Starobinsky R^2 (red, dashed), and Kallosh–Linde benchmarks $3\alpha = 1, 7$ (black, dotted). Stars mark the Branch-A values for the Planck 2018 and P-ACT-LB central n_s (shaded vertical bands: 68% CL). The horizontal line indicates the LiteBIRD sensitivity $\sigma(r) \approx 10^{-3}$.

The measured (projected) mass operator is then

$$\hat{M}_{\rm obs} = \mathcal{P}\left(\hat{M}_0 + \hat{V}_{\rm gluon}\right)\mathcal{P}^\dagger. \tag{11}$$

The operator $\mathcal{P}$ projects onto a proper subspace $\mathcal{H}_{\rm obs} \subsetneq \mathcal{H}$ and is therefore not invertible: $\mathcal{P}^{-1}$ does not exist, and $\mathcal{P}\mathcal{P}^\dagger \neq \mathbb{1}$. This non-unitarity is essential—a unitary (similarity) transformation would preserve the spectrum and produce no mass defect; the mass shift is a consequence precisely of the projection discarding the bare complex structure. The expectation value for a bound hadronic system is

$$M^{(\rm obs)} = \left\langle \hat{M}_{\rm obs} \right\rangle = M^{(\rm bare)} + M_{\rm gluon}. \tag{12}$$

From QCD lattice calculations and the trace anomaly, the gluonic contribution to the nucleon mass is approximately

$$\frac{M_{\rm gluon}}{M_{\rm nucleon}} \approx 0.98. \tag{13}$$

This ratio holds at the level of individual hadrons. Its projection to macroscopic gravitating objects (such as the Sun) requires additional input: the Schwarzschild mass of an astronomical body includes contributions from thermal kinetic energy, electrostatic and gravitational self-energy, and other binding terms beyond the bare quark–gluon substructure. Consequently equation (13) cannot be directly transcribed to $M_\odot^{(\mathrm{bare})} \approx 0.02\, M_\odot^{(\mathrm{obs})}$ without a derivation of how each contribution transforms under $\mathcal{P}$. We note this as an open problem: a precise macroscopic statement requires identifying the action of $\mathcal{P}$ on the gravitational and thermal degrees of freedom, which is beyond the scope of this Letter.

Consequence (nucleon level). At the hadronic level, the classical observer never measures the bare complex structure. Instead, the measured mass always contains the full gluonic dressing. The difference

$$\Delta M := M^{(\mathrm{obs})} - M^{(\mathrm{bare})} = M_{\mathrm{gluon}} \tag{14}$$

is not a fine-tuning; it is the footprint of the observer projection $\varphi(\tau) = \ln \tau_2$ acting on the QCD trace anomaly, *under the reinterpretation proposed above*—not an independently derived result.

What would make this more than a relabeling. A non-trivial version of this claim would predict a mass shift not already contained in Eq. (13)—for instance, a correction that depends on the modular parameter τ of some auxiliary structure, in a way that could in principle disagree with the lattice value. No such prediction is currently derived. Absent one, this section should be read as an interpretive remark, not as independent evidence for the τ-geometry framework.

8 Discussion

The logical chain of this Letter is:

1. M0 (structural realism): observables are functions on isomorphism classes.
2. Complex time: Postulate A1, motivated by the uniqueness of $\mathbb{C}$ for modular structure.
3. Photonic topology: Postulate A2, motivated by the moduli-space dimension count ($g = 1$ unique).
4. Modular invariance: Theorem 1, derived from M0 applied to elliptic-curve moduli.
5. $K = -1$: shape fixed by invariance; scale fixed by the explicit Postulate N—the framework's only continuous input.
6. Field-space identification: Postulate A4 (new in v6), identifying the inflaton field space with the time plane; flagged as the framework's least motivated postulate.
7. $\alpha = 2/3$: from $K_{\mathrm{fields}} \equiv K$, given the identification of Postulate A4.
8. Predictions: branch A, $r = 2(1 - n_s)^2$; branch B, $n_s \simeq 1 - 2/N_e$ with $r = O(10^{-5})$.

The framework requires no anthropic reasoning and no appeal to specific compactifications; where existing $SL(2, \mathbb{Z})$ cosmologies select α through string embeddings [6], the present proposal selects it through the intrinsic geometry of the modular curve, at the price of two explicitly stated inputs: the normalization postulate N and the field-space identification postulate A4. Its value rests on four things: the theorem status of modular invariance, the explicitness of its continuous input, the explicitness of its structural input, and the observationally decidable fork of Section 5.

8.1 Epistemological robustness: consistency relation versus fixed point

The recent movement of the n_s constraints — from Planck 2018 (0.9649) through CMB-SPA (0.9684) to BAO-inclusive combinations (0.973–0.974) — illustrates a methodological point about what this framework does and does not predict. Analyses of the shifted constraints [14] report that CMB-SPA combined with DESI disfavors, at more than 2σ, the models preferred by Planck alone (Higgs inflation, Starobinsky R^2, exponential α-attractors), in favor of e.g. polynomial α-attractors. Such statements are made against models read as *points* in the (n_s, r) plane at a fixed fiducial N_e. The primary falsifiable content of the present framework is not a point but the one-dimensional relation (4): a shift in the measured n_s moves the predicted r along the curve $r = 2(1-n_s)^2$ without touching the relation itself, which is why the data tables of Sections 4.4 and 6 update cleanly under each new data set. This robustness has a precise and limited scope, and we state its limit explicitly: it holds for the r-prediction channel only. Through $n_s \simeq 1-2/N_e$ the framework inherits the same N_e tension as the rest of the asymptotic α-attractor class, and if BAO-inclusive n_s values persist after the 2.8σ CMB–BAO tension is resolved, the required $N_e \approx 74$–78 must be accounted for by reheating history or by corrections to (1) — an explanatory debt the consistency-relation framing defers but does not cancel. The claim is therefore comparative, not absolute: under shifting n_s constraints, a consistency relation degrades more gracefully than a fixed-point prediction, because the datum that falsifies it is a joint measurement of (n_s, r) off the curve, not a drift in n_s alone.

8.2 Outlook: gravity as an entropic projection

The observer projection $\mathcal{P} : \mathcal{H} \to \mathcal{H}_{\mathrm{obs}}$ is non-unitary ($\mathcal{P}\mathcal{P}^\dagger \neq \mathbb{1}$) and therefore generates information loss or, equivalently, entropy. In Jacobson's thermodynamic derivation [21] and in Verlinde's entropic-gravity programme [22, 23], an entropy gradient on a local holographic screen induces an effective gravitational force, and the Einstein field equations emerge as an equation of state.

A natural-looking candidate screen within τ-geometry is the boundary of the fundamental domain $\mathcal{F}$, whose hyperbolic area $\mathrm{Area}(\mathcal{F}) = \pi/3$ (Sec. 4.2) is a fixed invariant of the moduli orbifold $X(1)$, paired with the entropy generated by $\mathcal{P}$ along the geodesic $\mathrm{Re}\,\tau = 0$,

$$\Delta S = \ln \frac{\dim \mathcal{H}}{\dim \mathcal{H}_{\mathrm{obs}}}. \tag{15}$$

Direct examination shows this candidate fails. Jacobson's derivation requires the horizon entropy to vary along the affine parameter of the local null congruence at each spacetime point, so that its variation reproduces the local curvature term entering the construction. Both $\mathrm{Area}(\mathcal{F})$ and ΔS above are constants, independent of the spacetime point and of the null direction, because $\mathcal{F}$ is a moduli-space (field-space) object, not a spacetime one. Substituting a constant entropy into the Clausius relation $\delta Q = T\delta S$ forces $T_{ab}k^a k^b = 0$ for every null k^a at every point; a symmetric tensor with this property is proportional to the metric, $T_{ab} = \lambda(x) g_{ab}$ (since $g_{ab}k^a k^b = 0$ trivially too), not in general zero. The forced condition is thus a pure vacuum equation of state, $p = -\rho$, at every point—in conflict with the kinetic term of Eq. (5), whose gradient contribution $\partial_\mu \tau_i \partial_\nu \tau_j \delta^{ij}$ is not in general proportional to $g_{\mu\nu}$ whenever $\dot{\tau} \neq 0$, precisely the regime of the reheating oscillation this outlook is meant to describe.

A screen that does supply the required local variation is the ordinary spacetime causal horizon used in Jacobson's original construction, with the standard Bekenstein–Hawking density $\eta = 1/4\ell_P^2$; with that choice the argument goes through and reproduces the Einstein equations sourced by the τ-field stress tensor of Section 5. This is, however, Jacobson's theorem applied to this framework's matter content—true for any local field theory satisfying the equivalence principle—and not an independent test of the modular structure proposed here. Whether the mass defect of Section 7 is related to this mechanism at all remains open. We no longer regard the entropic route as a promising source of new, falsifiable content; this subsection is retained to record the check and to correct the screen choice for anyone completing the argument sketched in an earlier version of this note.

Separately, the symmetric tensor product of two spin-1 photon states (Postulate A2) decomposes as $1 \otimes_{\rm sym} 1 = 0 \oplus 2$, and the spin-2 sector is the correct representation for a massless graviton. This observation about polarization content is independent of the entropic-screen question above and does not by itself establish a derivation of the Einstein–Hilbert action

$$S_{\rm grav} = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g}\, R. \tag{16}$$

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Version note (v8, erratum 2026-07-05)

Subsection 8.2 is corrected. A direct check of the entropic-gravity outlook (Jacobson's construction [21] applied with the fundamental-domain boundary $\partial\mathcal{F}$ as holographic screen) shows it fails: both the candidate area $\text{Area}(\mathcal{F}) = \pi/3$ and the candidate entropy ΔS are constants independent of the spacetime point, so the Clausius relation forces T_{ab} into a pure vacuum form $T_{ab} = \lambda(x) g_{ab}$ (a symmetric tensor vanishing on all null vectors is proportional to the metric, not in general zero)—in conflict with the kinetic term of Eq. (5) whenever $\dot{\tau} \neq 0$. The corrected screen (the ordinary spacetime causal horizon, with the standard Bekenstein–Hawking density) is consistent but reduces to Jacobson's theorem applied to this framework's matter content, adding no independent test. No other section is affected; the r and mass-defect predictions of Sections 4–7 are untouched.

Version note (v7 → v8)

A new Discussion subsection 8.1 ("Epistemological robustness: consistency relation versus fixed point") is added: it argues, with reference to the shifted n_s constraints [14], that the framework's primary falsifiable content is the one-dimensional relation $r = 2(1-n_s)^2$ rather than a point in the (n_s, r) plane at fixed N_e, and states explicitly the limit of this robustness claim: it applies to the r-prediction channel only, while the N_e tension is inherited through $n_s \simeq 1 - 2/N_e$ and is deferred, not cancelled. The entropic-gravity outlook is renumbered to subsection 8.2. No other changes.

Version note (v6 → v7)

(i) Section 4.4: two rows added to the data table — CMB-SPA (SPT-3G D1 + Planck + ACT DR6; $n_s = 0.9684 \pm 0.0030$, hence $r = 0.0020 \pm 0.0004$, $N_e \approx 63$) and CMB-SPA + DESI BAO ($n_s = 0.9728 \pm 0.0027$, hence $r = 0.0015 \pm 0.0003$); a paragraph notes that SPT-3G shifts the CMB-only n_s downward (softening the N_e tension to $N_e \approx 63$), that the strong upward pull on n_s is BAO-driven, and that CMB and DESI BAO are themselves in 2.8σ tension within ΛCDM, so the CMB-only and BAO-inclusive rows bracket the prediction. (ii) Section 6: a middle column

($n_s = 0.9684$) added to the model table; a new paragraph "Experimental landscape (as of mid-2026)" records the July 2025 cancellation of CMB-S4 (the ratio test $r_A/r_{\rm Star} = 2/3$ loses its funded carrier), the LiteBIRD replanning toward a JFY2036 launch (branch discrimination shifts to the late 2030s), and the near-term role of Simons Observatory ($\sigma(r) \approx 0.003$) for the upper branch-A band; the falsification criteria are annotated accordingly. (iii) Abstract updated to match. (iv) Six references added (SPT-3G D1 spectra and B-modes, n_s-spectrum analysis, Simons Observatory forecasts, CMB-S4 statement, LiteBIRD status). (v) No changes to Sections 1–3, 5, 7, 8, or to the framework's postulates and derivations.

Version note (v5 $\to$ v6)

(i) A new Postulate A4 makes explicit the previously implicit identification of the inflaton field space with the modular time plane (Sec. 2.5), replacing the informal "by economy" justification of v5's Discussion; A4 is flagged as the framework's least-motivated postulate, and a sentence naming this explicitly is added to the Discussion. (ii) Section 7 is retitled "Observer projection as a reinterpretation of the mass defect" and reframed: an opening paragraph states that the numerical content is imported from lattice QCD and not derived, and a closing paragraph states explicitly that the section makes no new numerical prediction. (iii) No changes to Sections 1–6 (inflationary predictions, Table, Fig. 1) or to Section 8.1.

Version note (v4 $\to$ v5)

(i) Section 7 revised: the extrapolation from the nucleon-level gluonic fraction (13) to a macroscopic Schwarzschild mass defect is identified as an open problem requiring explicit treatment of thermal, gravitational, and electromagnetic contributions to the projected mass; the nucleon-level consequence is retained. (ii) Section 8.1 added: a speculative entropic-gravity outlook connecting the non-unitary projection $\mathcal{P}$ to the frameworks of Jacobson and Verlinde; the spin-2 content of the symmetric photon tensor product is noted as the appropriate graviton representation. (iii) Three new references added (Jacobson 1995, Verlinde 2011, Verlinde 2017). (iv) Abstract and table of contents updated.

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